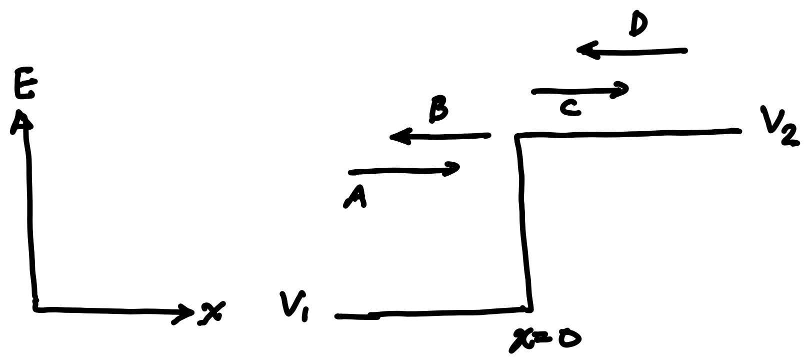
Transmission & Reflection



Consider a particle with energy E approaching the potential barrier.

$$-\frac{\hbar^2}{2m}\psi'' + V\psi = E\psi \quad \Rightarrow$$

$$\begin{cases} \psi_1'' = -\overbrace{\dfrac{2m(E-V_1)}{\hbar^2}}^{k_1^2}\psi_1 \to \psi_1 = Ae^{ik_1x} + Be^{-ik_1x} & x<x_0 \\[2ex] \psi_2'' = -\overbrace{\dfrac{2m(E-V_2)}{\hbar^2}}^{k_2^2}\psi_2 \to \psi_2 = Ce^{ik_2x} + De^{-ik_2x} & x \geq x_0 \end{cases}$$

Apply BC's ψ & ψ' are continous at $x=0$

$$\begin{cases} \psi_1(0) = \psi_2(0) \to A+B = C+D \\[1ex] \psi_1'(0) = \psi_2'(0) \to k_1A - k_1B = k_2C - k_2D \to A-B = \dfrac{k_2}{k_1}C - \dfrac{k_2}{k_1}D \end{cases}$$

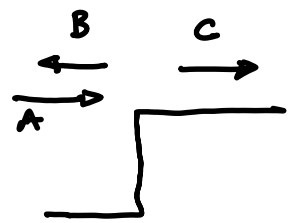
Suppose we know that:

$$\begin{cases} \text{① the particle is approaching from the left: } |A|^2 = 1 \\ \text{② There is no potential change at } x>0 \text{ : } |D|^2 = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 1 + B = C \\ 1 - B = \frac{k_2}{k_1} C \end{cases} \Rightarrow \begin{cases} B = \dfrac{1 - k_2/k_1}{1 + k_2/k_1} \\ C = \dfrac{2}{1 + k_2/k_1} \end{cases}$$

$$P = \hbar k = mv \rightarrow v = \frac{\hbar k}{m} \Rightarrow$$

$$\begin{cases} |B|^2 = \left(\dfrac{1 - k_2/k_1}{1 + k_2/k_1}\right)^2 = \left(\dfrac{v_1 - v_2}{v_1 + v_2}\right)^2 \\ |C|^2 = \dfrac{4}{(1 + k_2/k_1)^2} = \dfrac{4}{(1 + v_2/v_1)^2} \end{cases}$$



Let's see if $|C|^2 >$ or < 1 :

$$\begin{cases} k_1^2 = \dfrac{2m(E - V_1)}{\hbar^2} \\ k_2^2 = \dfrac{2m(E - V_2)}{\hbar^2} \end{cases}$$

Since $V_2 > V_1 \Rightarrow k_2 < k_1 \Rightarrow v_2 < v_1$

$$\Rightarrow \left(1 + \frac{v_2}{v_1}\right)^2 < 2^2 \Rightarrow |C|^2 > 1$$

Compare this with $|A|^2 = 1$: this means that the particle is more likely to be at $x > 0$ than at $x < 0$ eventhough $V_1 < V_2$ (potentials). So doesn't the particle have the preference to go to minimum energy?

Since $v_1 > v_2$, the particle spends more time at region 2 than 1. So it is more likely to find it in region 2.

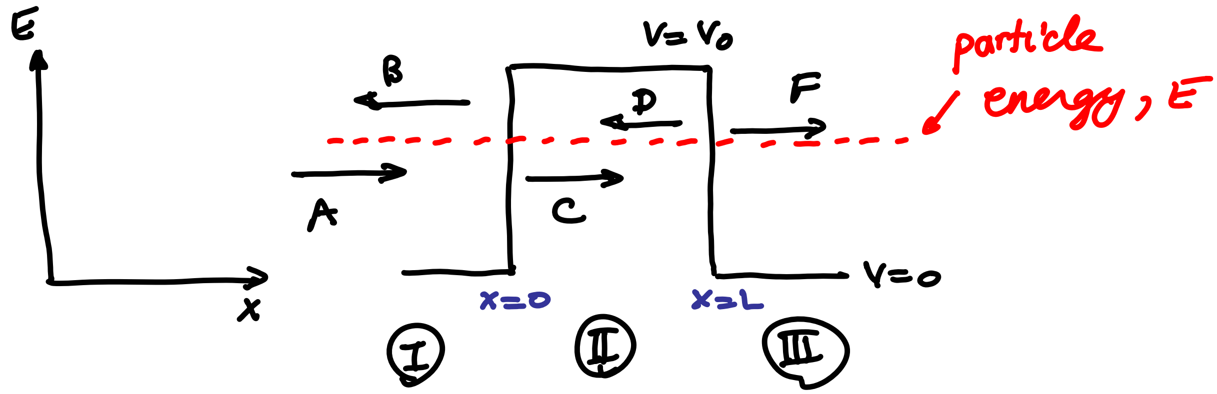
Be careful:

It is more accurate to speak about current density when we have moving particle like in the previous problem. So we should in fanct compare J_1 & J_2 and not $|A|^2$ & $|C|^2$.

Tunneling

Tunneling is fundamentally quantum mechanical.

Consider a particle with $E < V_0$ approaching a potential barrier of height V_0:



In region I: $\psi_1(x) = Ae^{ikx} + Be^{-ikx}$

II: $\psi_2(x) = Ce^{Kx} + De^{-Kx}$

III: $\psi_3(x) = Fe^{ikx}$

Of course as usual: $E = \dfrac{\hbar^2 k^2}{2m}$ Kinetic energy

$$V_0 - E = \frac{\hbar^2 K^2}{2m}$$

B.C.

(1) $\psi_1 = \psi_2$ at $x=0 \rightarrow A + B = C + D$

(2) $\psi_1' = \psi_2'$ at $x=0 \rightarrow ikA - ikB = CK - DK$

(3) $\psi_2 = \psi_3$ at $x=L \rightarrow Ce^{KL} + De^{-KL} = Fe^{ikL}$

(4) $\psi_2' = \psi_3'$ at $x=L \rightarrow CKe^{KL} - DKe^{-KL} = ikFe^{ikL}$

we're looking for the tunneling transmission probability:

$\left|\frac{F}{A}\right|^2$. So let's eliminate the other constants:

$(1) + \frac{(2)}{ik} \rightarrow \quad 2A = \left(1 - \frac{ik}{K}\right) C + \left(1 + \frac{ik}{K}\right) D \qquad (5)$

$(3) + \frac{(4)}{K} \rightarrow \quad 2C e^{KL} = \left(1 + \frac{ik}{K}\right) F e^{ikL} \qquad (6)$

$(3) - \frac{(4)}{K} \rightarrow \quad 2D e^{-KL} = \left(1 - \frac{ik}{K}\right) F e^{ikL} \qquad (7)$

Substitute (6) & (7) in (5) $\Rightarrow$

$$2A = \left(1 - \frac{ik}{k}\right)\left(1 + \frac{ik}{K}\right) \frac{F}{2} e^{ikL - KL} + \left(1 - \frac{ik}{k}\right)\left(1 - \frac{ik}{K}\right) \frac{F}{2} e^{ikL + KL}$$

Simplify to get:

$$\frac{F}{A} = \frac{i4k/K}{\left[-\left(1 - \frac{ik}{K}\right)^2 e^{KL} + \left(1 + \frac{ik}{K}\right)^2 e^{-KL}\right]} e^{ikL}$$

$$\left|\frac{F}{A}\right|^2 = \left(\frac{F}{A}\right)\left(\frac{F^*}{A^*}\right)$$

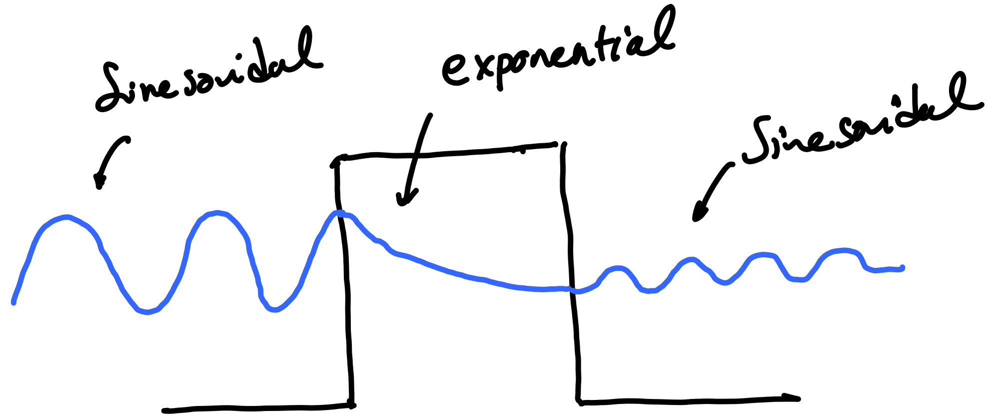
$$= \frac{\left(\frac{i4k}{K}\right)\left(\frac{-i4k}{K}\right)}{\left[-\left(1 - \frac{ik}{K}\right)^2 e^{KL} + \left(1 + \frac{ik}{K}\right)^2 e^{-KL}\right]\left[-\left(1 + \frac{ik}{K}\right)^2 e^{KL} + \left(1 - \frac{ik}{K}\right)^2 e^{-KL}\right]}$$

Do the math and get: (See p. 147 book)

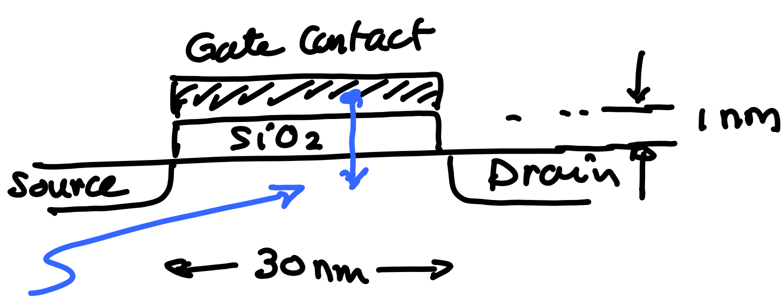
$$\left|\frac{F}{A}\right|^2 = \frac{1}{1+\frac{k^2+K^2}{2kK}\sinh^2(KL)}$$

Tunneling transmission probability

(note: k & K are related to E and V_0)



Electron tunneling is limitting us to make smaller CMOS transistors:



electron tunnels through the gate oxide!